# Pseudospin entanglement and Bell test in graphene

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We propose a way of producing and detecting pseudospin entanglement between electrons and holes in graphene. Electron-hole pairs are produced by a fluctuating potential and their entanglement is demonstrated by a current correlation measurement. The chirality of electrons in graphene facilitates a well-controlled Bell test with (pseudo-)spin projection angles defined in *real* space.

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## I. INTRODUCTION

The entanglement between internal degrees of freedom of an electron and a hole in the Fermi sea is of both fundamental and practical interest. It has been recognized as a form of entanglement that does not require many-body interactions[1] and comparatively simple ways of generating it experimentally have been proposed.[1,2] Yet, the experimental demonstration of electron-hole entanglement in solid-state structures is still outstanding.

Very recently Neder *et al.*[3] have implemented a quantum Hall interferometer that had been proposed[2] for the generation and detection of electron-hole entanglement. This interferometer has allowed the observation of the "two-particle interference" that is at the core of electron-hole entanglement.[3] While this is a first indication of entanglement production in the experiment of Neder *et al.*, a conclusive verification of that entanglement has not yet been achieved. It has been proposed[1,2,4,5] that the generated entanglement is best verified through the violation of a Bell inequality—an inequality between correlators of spin degrees of freedom whose violation is an unambiguous signature of entanglement and contradicts a classical intuition called "local realism." Experimentally, the demonstration of such a violation, however, meets with significant challenges. First, the decoherence in the interferometer of Ref. 3 needs to be reduced significantly in order to safely preserve the entanglement from the time of its production to its detection. In addition, a test of Bell inequalities requires measurements of spin-1/2 degrees of freedom along variable quantization axes. In the setup of Ref. 3 these quantization axes are defined by scattering amplitudes that are poorly controlled experimentally, requiring an implementation through trial and error.

In this paper we put forward a theoretical proposal for the generation and detection of electron-hole entanglement that does not suffer from the above mentioned problems. We propose to entangle the pseudospin[7] degree of freedom of electrons and holes in graphene[8–10] by means of the "pumping" mechanism of Refs. 11 and 12. The typical energy scales in graphene are considerably higher than those in GaAs, which has for instance allowed an observation of the quantum Hall effect at room temperature.[13] At comparable temperatures one thus expects the decoherence in graphene to be much weaker than in the experiment of Ref. 3, addressing the first of the above issues. We show below that entanglement in graphene should be observable at temperatures at least an order of magnitude higher than those in the experiment of Ref. 3.

In addition, we formulate a Bell test through current correlation measurements that overcomes the mentioned problems of previously pursued entanglement detection schemes.[1,2,4,5] Electrons in graphene have a definite chirality (for a certain band-structure "valley"), moving in the direction of their pseudospin. The pseudospin of an excitation can thus be measured through its direction of motion. This affords a Bell test with straightforward and transparent control of the (pseudo)spin-quantization axes that are now defined in *real* space. The Bell test proposed in Refs. 1, 2, 4, and 5 is only valid in the regime of temperatures $T$ that are low compared to the voltage $V$ applied to the interferometer: $kT \ll eV$. Its application at finite temperatures faces a problem that has been discussed recently in Ref. 6. The authors of Ref. 6 suggest a cure of that issue whose experimental implementation, however, is challenging; it requires the addition of resonant levels to the setup. Here we avoid the problem pointed out in Ref. 6 by a suitable postselection of the entangled electron-hole pairs. That selection is implemented simply by subtracting the thermal background from all measured current correlators.

This proposal thus avoids some of the main difficulties of previous attempts to generate electron-hole entanglement. The more challenging requirements in the present proposal are a relatively localized infrared light source and a ballistic graphene sample.

## II. SETUP

We consider a sheet of ballistic graphene at low temperature $T$ and nonzero Fermi energy $\varepsilon_{\mathrm{F}} \gg kT$ in a vanishing magnetic field $B$. The sheet is well coupled to an electron reservoir along its rim, as shown in Fig. 1. We formulate the low-energy Hamiltonian of graphene in single-valley form through a unitary transformation that renders the Dirac model valley isotropic,[14]

$$H_0 = v\boldsymbol{\sigma}\cdot\boldsymbol{p}. \tag{1}$$

Here, $\boldsymbol{\sigma}=(\sigma_x,\sigma_y)$ is a vector of Pauli matrices in pseudospin space, $\boldsymbol{p}$ is the electron momentum, and $v$ is the Fermi velocity. The graphene sheet is subject to a local fluctuating potential, as described by the Hamiltonian,

 

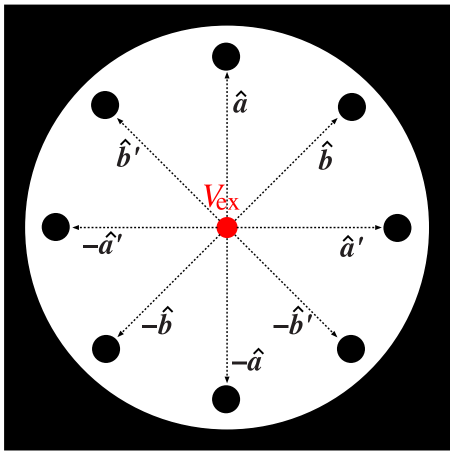


FIG. 1. (Color online) Proposed setup: a localized fluctuating electric potential $V_{\rm ex}$ produces entangled electron-hole pairs in the center of a graphene sheet. The excitations are either drained at the rim of the sheet or leave into tunnel contacts at locations $r_\alpha \hat{\boldsymbol{\alpha}}$ (where $\alpha$ takes the values $\pm a$, $\pm a'$, $\pm b$, or $\pm b'$). The generated entanglement has signatures in correlations between the currents into these tunnel contacts. The starlike setup that is shown allows the maximal violation of a Bell inequality.

$$H_{\rm ex}(t) = \int d\boldsymbol{x} u(\boldsymbol{x}) e V_{\rm ex}(t) \vec{\psi}^\dagger(\boldsymbol{x}) \cdot \vec{\psi}(\boldsymbol{x}). \tag{2}$$

Here we have switched to second-quantized notation with electron annihilation operators $\vec{\psi}$ that are vectors in pseudospin space. The fluctuating potential is focused on a small region of spatial extent $l_{\rm ex}$ in the middle of the graphene sheet and it is centered around the origin of our coordinate system. Its shape is given by a function $u$ that is normalized to $k_{\rm F}^2 \int d\boldsymbol{x} u(\boldsymbol{x}) = 1$ [for instance $u(\boldsymbol{x}) = \exp(-|\boldsymbol{x}|^2/2l_{\rm ex}^2)/2\pi k_{\rm F}^2 l_{\rm ex}^2$]. The length $l_{\rm ex}$ is assumed to be large compared to the lattice spacing $l_{\rm lattice}$, but small compared to the Fermi wavelength $2\pi/k_{\rm F}$ (which, in a clean system, can be chosen at will by tuning the Fermi energy), that is $l_{\rm lattice} \ll l_{\rm ex} \ll 2\pi/k_{\rm F}$. We assume that the frequency spectrum of the potential correlator $c_V(-\omega) = \int dt \exp(-i\omega t)\langle V_{\rm ex}(t) V_{\rm ex}(0)\rangle$ is relatively flat until it vanishes above a high-frequency cutoff $\omega \approx \Omega$ with $kT \ll \Omega \ll \varepsilon_{\rm F}$.[11,12,15] The fluctuating potential $V_{\rm ex}$ creates pairs of electrons and holes (in the sense of an electron missing in an otherwise filled Fermi sea) that propagate outward before they are reflectionlessly drained by the reservoir surrounding the graphene sheet. On their way to the rim they are able to leave through tunnel contacts $\alpha$ with sizes $l_\alpha \ll r_\alpha$ into additional electron reservoirs at locations $\boldsymbol{x} = r_\alpha \hat{\boldsymbol{\alpha}}$, where $|\hat{\boldsymbol{\alpha}}| = 1$ and $r_\alpha \gg 2\pi/k_{\rm F}$ (see Fig. 1),

$$H_{{\rm T},\alpha} = \int d\boldsymbol{x} \vec{\psi}^\dagger(\boldsymbol{x}) \cdot \begin{pmatrix} w_\alpha^{\rm A}(\boldsymbol{x}) \\ w_\alpha^{\rm B}(\boldsymbol{x}) \end{pmatrix} \psi_\alpha^{\rm res} + {\rm H.c.} \tag{3}$$

(Ref. 16). Here, the functions $\vec{w}_\alpha$ are centered around $\boldsymbol{x} = r_\alpha \hat{\boldsymbol{\alpha}}$ and the operator $\psi_\alpha^{\rm res}$ annihilates electrons in the reservoir of contact $\alpha$. All electron reservoirs are in thermal equilibrium with the graphene sheet. Every tunnel contact $\alpha$ has one counterpart $-\alpha$ in direction $-\hat{\boldsymbol{\alpha}}$.

## III. ENTANGLEMENT PRODUCTION

We first consider the excitations created by a short potential pulse, $eV_{\rm ex}(t) = \zeta\delta(t - t_{\rm ex})$. In first-quantized form the low-energy contribution ($|\boldsymbol{p} - \boldsymbol{p}'| \ll k_{\rm F}$) to the electron-hole pair that is produced at first order in $\zeta$ reads (we set $\hbar = 1$),

$$|\psi(t)\rangle_\zeta \big|_{t=t_{\rm ex}+0^+} = \zeta \sum_{pp'} |\boldsymbol{p}\rangle^{\rm el} |\boldsymbol{p}'\rangle^{\rm h} (|\uparrow\rangle^{\rm el}|\uparrow\rangle^{\rm h} + |\downarrow\rangle^{\rm el}|\downarrow\rangle^{\rm h}), \tag{4}$$

where $\uparrow$ and $\downarrow$ specify the pseudospin direction and $|\,\rangle^{\rm el}$ and $|\,\rangle^{\rm h}$ are electron and hole amplitudes, respectively (with the convention that a hole has the same pseudospin as the electron that it replaces). The pseudospins of the electron and the hole described by $|\psi\rangle_\zeta$ [Eq. (4)] are entangled. They form a so-called Bell pair. A source with a periodically varying potential $V_{\rm ex}(t)$ serves as a steady supply of such Bell pairs. Excitations that appear at higher order in $\zeta$ are negligible if $|eV_{\rm ex}(t)| \ll \varepsilon_{\rm F}$ for all $t$, which we assume henceforth (this is not realizable for the sharply peaked time dependence chosen for the illustrative argument above, but it can be achieved for any realistic oscillatory time dependence).

## IV. ENTANGLEMENT DETECTION

The Heisenberg equations of motion corresponding to the Hamiltonian $H_0$, Eq. (1),

$$\dot{\boldsymbol{p}} = 0, \quad \dot{\boldsymbol{\sigma}} = 2v\boldsymbol{p} \times \boldsymbol{\sigma}, \quad \dot{\boldsymbol{x}} = v\boldsymbol{\sigma}, \tag{5}$$

show that the pseudospin of an electron is not conserved. We therefore postselect orbital states of the form $|p\rangle_\alpha = (|p\hat{\boldsymbol{\alpha}}\rangle + |-p\hat{\boldsymbol{\alpha}}\rangle)/\sqrt{2}$. The initial wave function $|\psi\rangle_\zeta$ [Eq. (4)] factorizes into an isotropic orbital part and a pseudospin part. It follows that at $t = t_{\rm ex}$ also the pseudospin state of all postselected electron-hole pairs is given by the pseudospin part of Eq. (4) and entangled. Moreover, the (un-normalized) density matrix of a postselected electron $\rho_{pp'}^\alpha = \langle p|_\alpha \rho |p'\rangle_\alpha$, $\rho$ being the density matrix of the electron before postselection, takes the form $\rho_{pp'}^\alpha(t) = \cos[2vp(t - t_{\rm ex})]\cos[2vp'(t - t_{\rm ex})]\rho_{pp'}^\alpha(t_{\rm ex})$. After normalization $\rho^\alpha$ is time independent. The pseudospins of the postselected excitations are thus conserved and so is their entanglement. We propose to verify that entanglement by violation of a Bell inequality. This requires a measurement of the postselected pseudospins with variable quantization axes. The tunnel contacts $\alpha$ serve that purpose. To see this we integrate Eq. (5) to find

$$\boldsymbol{x}(t) = \boldsymbol{x}(t_{\rm ex}) + v(t - t_{\rm ex}) \frac{[\boldsymbol{p}(t_{\rm ex}) \cdot \boldsymbol{\sigma}(t_{\rm ex})]\boldsymbol{p}(t_{\rm ex})}{|\boldsymbol{p}(t_{\rm ex})|^2} + \mathcal{O}\left(\frac{1}{|\boldsymbol{p}|}\right). \tag{6}$$

Consider an electron (before postselection) that is produced at $t = t_{\rm ex}$ and $\langle \boldsymbol{x}(t_{\rm ex})\rangle = 0$, such that semiclassically the first term in Eq. (6) vanishes. The third, oscillatory term in Eq. (6) is smaller than the second one by a factor $(k_{\rm F} r_\alpha)^{-1}$ and negligible in our limit. Assume that this electron is detected in contact $\alpha$ at time $t$, such that $\boldsymbol{x}(t) = r_\alpha \hat{\boldsymbol{\alpha}}$. This projects the initial state of the electron onto eigenstates of $\boldsymbol{x}(t)$ [Eq. (6)] with eigenvalue $r_\alpha \hat{\boldsymbol{\alpha}}$. To our accuracy, when only the second

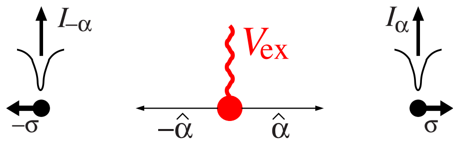


FIG. 2. (Color online) Pseudospin measurement: electron and hole excitations are generated by $V_{\text{ex}}$ at $\boldsymbol{x}=0$. They contribute to the current $I_\alpha$ into contact $\alpha$ at $\boldsymbol{x}=r_\alpha\hat{\boldsymbol{\alpha}}$ if their velocity satisfies $\dot{\boldsymbol{x}}\parallel\hat{\boldsymbol{\alpha}}$. The pseudospin $\boldsymbol{\sigma}$ of these excitations is fixed due to their chirality: one has $\boldsymbol{\sigma}\parallel\dot{\boldsymbol{x}}$ and $\boldsymbol{\sigma}$ thus points "up" along the quantization axis $\hat{\boldsymbol{\alpha}}$. The opposing contact $-\alpha$ at $\boldsymbol{x}=-r_\alpha\hat{\boldsymbol{\alpha}}$ collects excitations with pseudospin "down" along the same axis.

term in Eq. (6) is relevant, such a measurement projects onto amplitudes with $\boldsymbol{p}\parallel\hat{\boldsymbol{\alpha}}$. Moreover, substituting $\boldsymbol{p}(t_{\text{ex}})/|\boldsymbol{p}(t_{\text{ex}})|=\hat{\boldsymbol{\alpha}}$ into Eq. (6) (the sign of $\boldsymbol{p}(t_{\text{ex}})$ is arbitrary since in enters quadratically), we see that a measurement of an electron in contact $\alpha$ projects onto amplitudes with $\boldsymbol{\sigma}(t_{\text{ex}})\cdot\hat{\boldsymbol{\alpha}}=1$. Likewise, detection of an electron in contact $-\alpha$ projects onto amplitudes with $\boldsymbol{\sigma}(t_{\text{ex}})\cdot\hat{\boldsymbol{\alpha}}=-1$.[17] We conclude that the tunnel contacts $\alpha$ collect currents $I_\alpha$ of electrons and holes with a definite initial pseudospin when measured along quantization axes $\hat{\boldsymbol{\alpha}}$ that are defined by the locations of the contacts $\alpha$ in real space, as illustrated in Fig. 2. As discussed above, the initial pseudospin state of the created electron-hole pairs before postselection equals the pseudospin state of any of the postselected electron-hole pairs. Also the pseudospin of the postselected pseudospins may thus be inferred from a measurement of the currents $I_\alpha$.

In order to demonstrate the entanglement of the postselected electron-hole pairs we formulate a slightly modified Clauser-Horne-Shimony-Holt inequality,

$$\mathcal{B}\leq 2 \quad \text{with} \quad \mathcal{B}=C_{ab}+C_{a'b}+C_{ab'}-C_{a'b'}, \tag{7}$$

in terms of symmetrized correlators,

$$C_{ab}=\frac{1}{2}\langle\psi|(\hat{\boldsymbol{a}}\cdot\boldsymbol{\sigma}^{\text{el}})(\hat{\boldsymbol{b}}\cdot\boldsymbol{\sigma}^{\text{h}})+(\hat{\mathbf{b}}\cdot\boldsymbol{\sigma}^{\text{el}})(\hat{\boldsymbol{a}}\cdot\boldsymbol{\sigma}^{\text{h}})|\psi\rangle, \tag{8}$$

of an electron pseudospin $\boldsymbol{\sigma}^{\text{el}}$ and a hole pseudospin $\boldsymbol{\sigma}^{\text{h}}$, projected onto the unit vectors $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$. Only entangled electron-hole pairs can violate the inequality (7) and the parameter $\mathcal{B}$ is thus an "entanglement witness."[18]

At zero temperature and in our limit of dilute electron-hole pairs $|eV_{\text{ex}}|\ll\varepsilon_{\text{F}}$ one shows along the lines of Refs. 4 and 5 that the parameter $C_{ab}$ can be expressed through zero-frequency current correlators. One needs to correlate the currents of electrons and holes with definite pseudospins when measured along the quantization axes $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$, respectively. The reasoning of the paragraph around Eq. (6) allows us to relate these currents to the tunnel currents $I_{\pm a}$ and $I_{\pm b}$ into the contacts $\alpha=\pm a$ and $\alpha=\pm b$. In case all tunnel contacts couple with the same strength to the relevant electrons or holes one concludes in this way that[4,5]

$$C_{ab}=\frac{\sum_{\sigma,\sigma'=\pm1}\sigma\sigma' c_{\sigma a,\sigma' b}}{\sum_{\sigma,\sigma'=\pm1}c_{\sigma a,\sigma' b}}, \tag{9}$$

where $\tilde{c}_{a,b}=\int dt\langle\delta I_a(t)\delta I_b(0)\rangle$ has to be substituted for $c_{a,b}$. Unlike in the proposals of Refs. 1 and 2, here not every produced electron-hole pair is detected. To find $c_{a,b}$ in the general case, when the tunnel coupling strengths to different reservoirs $\alpha$ are not equal, one therefore has to normalize the correlators $\tilde{c}_{a,b}$ by the detection probabilities $W_\alpha$,

$$c_{a,b}=(W_aW_b)^{-1}\int dt[\langle\delta I_a(t)\delta I_b\rangle-\langle\delta I_a(t)\delta I_b\rangle_{V_{\text{ex}}=0}]. \tag{10}$$

The probabilities $W_\alpha$ can be measured through the ac response of the currents $I_\alpha$ to the excitation potential $V_{\text{ex}}$ at frequencies $\omega\ll\min\{v/l_\alpha,\varepsilon_{\text{F}}\}$,

$$W_\alpha=\left|\frac{\varepsilon_{\text{F}}}{\omega}\right|\left|\frac{I_\alpha(\omega)}{e^2V_{\text{ex}}(\omega)}\right|. \tag{11}$$

Extra care has to be taken at finite temperature. We avoid the issue pointed out in Ref. 6 here by an additional postselection of the electron-hole pairs for which Eq. (7) is evaluated (see Appendix A for the details). This selection is made by the subtraction of the equilibrium current correlations in Eq. (10). We show in Appendix A that after that subtraction a violation of the Bell inequality [Eq. (7)] is an unambiguous signature of entanglement also at finite temperature, as long as $v/|r_a\hat{\boldsymbol{a}}\pm r_b\hat{\boldsymbol{b}}|\ll kT\ll\Omega$.

## V. PREDICTIONS

In our limit of small $\vec{w}_\alpha$ (that is tunneling contacts), $v/|r_a\hat{\boldsymbol{a}}\pm r_b\hat{\boldsymbol{b}}|\ll kT\ll\Omega\ll\varepsilon_{\text{F}}$, $|eV_{\text{ex}}|\ll\varepsilon_{\text{F}}$, $l_{\text{ex}}\ll 2\pi/k_{\text{F}}$, and $l_\alpha\ll r_\alpha$ we find (see Appendix B)

$$c_{a,b}=\frac{e^4}{2\varepsilon_{\text{F}}^2}(1+\hat{\boldsymbol{a}}\cdot\hat{\boldsymbol{b}})\int_{-\infty}^{\varepsilon_{\text{F}}}d\varepsilon\int_{\varepsilon_{\text{F}}}^{\infty}d\varepsilon' c_V(\varepsilon-\varepsilon'). \tag{12}$$

We conclude that the pseudospin correlators measured through Eqs. (9) and (10) take the form

$$C_{ab}=\hat{\boldsymbol{a}}\cdot\hat{\boldsymbol{b}}, \tag{13}$$

which is immediately shown to violate the modified CHSH inequality [Eq. (7)] for appropriate choices of the vectors $\hat{\boldsymbol{a}}$, $\hat{\boldsymbol{a}}'$, $\hat{\boldsymbol{b}}$, and $\hat{\boldsymbol{b}}'$. The maximal violation $\mathcal{B}=2\sqrt{2}$ for instance can be achieved in the symmetric starlike setup shown in Fig. 1, where the vectors $\hat{\boldsymbol{a}}'$, $\hat{\boldsymbol{b}}$, $\hat{\boldsymbol{a}}$, and $\hat{\boldsymbol{b}}'$ are separated by successive 45° angles.

For Eq. (13) to hold the created excitations must not change their pseudospin on the way to the tunnel contacts, for instance through decoherence. We thus assume excitation energies such that the inelastic mean-free path $l_{\text{in}}$ is long, $l_{\text{in}}\gg r_\alpha$. This condition is fulfilled at $\Omega\ll\min\{v_{\text{ph}}k_{\text{F}},\varepsilon_{\text{F}}/\sqrt{k_{\text{F}}r_\alpha}\}$, where $v_{\text{ph}}$ is the phonon velocity in graphene.[19,20] As anticipated, for suitable parameter values decoherence through inelastic processes in our proposal is already suppressed at energy scales much higher than those in the experiment of Ref. 3; for a Fermi wavelength of 50 nm (corresponding to a Fermi energy of $\varepsilon_F \approx 0.1$ eV) we find that coherence is maintained for excitation energies of up to $\Omega \ll 1$ meV at $r_\alpha \ll 1$ mm (limited by phonon scattering). The experiment of Ref. 3, in contrast, was performed at energies of $\approx 10$ $\mu$eV. We need in addition that also the elastic mean-free path $l_{\text{el}}$ is long, $l_{\text{el}} \gg r_\alpha$. Long elastic mean-free paths have been found in suspended sheets of graphene.[21] We conclude that the setup depicted in Fig. 1 allows to generate and conclusively demonstrate electron-hole entanglement if $\min\{v_{\text{ph}}k_F, \varepsilon_F/\sqrt{k_F r_\alpha}\} \gg \Omega \gg kT \gg v/|r_a\hat{\boldsymbol{a}} \pm r_b\hat{\boldsymbol{b}}|$.

## VI. DISCUSSION

We have proposed a way of creating and verifying pseudospin entanglement of electron-hole pairs in graphene. Bell pairs are produced by a fluctuating potential and their entanglement is demonstrated after postselection through violation of a Bell inequality. The quantization axes in the requisite pseudospin measurement are defined by the locations of tunnel contacts in real space. This simplicity of the pseudospin measurement is bought at a price: the postselected Bell pairs are not easily separated spatially since the advocated pseudospin measurement is nonlocal, as shown in Fig. 2. The produced Bell pairs, however, are entangled also with respect to their intrinsic spins,[22] which entanglement is readily spatially separated.[11,12] The proposed experiment is thus an intermediate step toward the generation and manipulation of spatially separated Bell pairs in electronic structures. Entanglement is generated by a mechanism that is able to produce spatially separated Bell pairs, but it is detected before that spatial separation is achieved.[23] The proposed detection mechanism affords three major advantages: (i) it suffers less from decoherence than previously pursued implementations of particle-hole entanglement; (ii) it allows a well-controlled Bell test with clearly defined (pseudo)spin-quantization axes; (iii) it avoids problems with earlier proposals of electron-hole entanglement detection at finite temperature. Our proposal thus overcomes some critical hurdles on the way to an observation of particle-hole entanglement in electronic structures.

## ACKNOWLEDGMENTS

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## APPENDIX A: BELL TEST

The relation between current correlations and the (pseudo)spin correlator $C_{ab}$ [Eq. (9)] has been proven in Refs. 4 and 5 at zero temperature in the limit of dilute electron-hole pairs (in the system discussed here that is $|eV_{\text{ex}}| \ll \varepsilon_F$). It has been pointed out recently[6] that extra care has to be taken at finite temperature. Thermal excitations in the reservoirs that collect the measured currents are then able to generate electron and hole currents that flow in the "wrong" direction: from the reservoirs into the system that contains the entangled electron-hole pairs. These currents pose a serious problem and they can invalidate a Bell test along the lines of Refs. 4 and 5 at finite temperature. In Ref. 6 an alternative, energy-selective detection scheme has been proposed that does not suffer from the same problem. Experimental implementations of energy-resolved detection, for instance through resonant levels,[26] are, however, expensive. In contrast to the situation studied in Ref. 6 the entangled electrons and holes in our proposal differ in energy. This affords a simpler cure of the problem.

The typical energy separation between the electron and the hole in the produced Bell pairs is $\Omega$. This allows us to restrict our attention by means of postselection to electron-hole pairs with electron energies $\varepsilon > \varepsilon_F + \omega$ and hole energies $\bar{\varepsilon} < \varepsilon_F - \omega$, where we choose $kT \ll \omega \ll \Omega$. Thermally activated electron-hole pairs in the graphene sheet and the reservoirs coupled to it have typical energies $|\varepsilon - \varepsilon_F|, |\bar{\varepsilon} - \varepsilon_F| \approx kT$. All but an exponentially suppressed number of them are excluded by the above postselection. This avoids the problem that has been pointed out in Ref. 6. At the same time our postselection includes almost all excitations created by $V_{\text{ex}}$ that have typical energies $|\varepsilon - \varepsilon_F|, |\bar{\varepsilon} - \varepsilon_F| \approx \Omega$. The above postselection may thus be implemented approximately by subtracting the statistical (thermal) contributions from all measured current correlators, leaving only correlations due to $V_{\text{ex}}$. These thermal correlations, in turn, may be inferred from a measurement of the respective correlators in equilibrium, at $V_{\text{ex}} = 0$. The above postselection should thus solve the problem pointed out in Ref. 6 with very moderate additional experimental effort: it is implemented by a second measurement in thermal equilibrium, as expressed in Eq. (10). Below we prove this expectation correct.

The two-particle density matrix after our postselection of electron-hole pairs in the conduction band with symmetrized orbital states $|p\rangle_\alpha^{\text{el}}$ and $|\bar{p}\rangle_{\bar{\alpha}}^{\text{h}}$ for electron and hole, respectively, at electron momenta $p > (\varepsilon_F + \omega)/v$ and hole momenta $\bar{p} < (\varepsilon_F - \omega)/v$ reads

$$\begin{aligned}\rho^{\text{el-h}\alpha\bar{\alpha}}_{\sigma\bar{\sigma},\sigma'\bar{\sigma}'}(p,\bar{p};p',\bar{p}') \propto{}& \Theta(vp - \varepsilon_F - \omega)\Theta(\varepsilon_F - \omega - v\bar{p}) \\ &\times \Theta(vp' - \varepsilon_F - \omega)\Theta(\varepsilon_F - \omega - v\bar{p}') \\ &\times \langle p|_\alpha^{\text{el}}\langle\sigma|^{\text{el}}\langle\bar{p}|_{\bar{\alpha}}^{\text{h}}\langle\bar{\sigma}|^{\text{h}}\rho_{\text{cond}} \\ &\times |p'\rangle_\alpha^{\text{el}}|\sigma'\rangle^{\text{el}}|\bar{p}'\rangle_{\bar{\alpha}}^{\text{h}}|\bar{\sigma}'\rangle^{\text{h}}, \end{aligned} \tag{A1}$$

[$\Theta(x)=1$ for $x>0$ and $\Theta(x)=0$ otherwise], where the pseudospins of the electron and the hole are denoted $\sigma$ and $\bar{\sigma}$, respectively. $\rho_{\text{cond}}$ is the density matrix of all electron-hole pairs in the conduction band of the graphene sheet (before postselection). Of all momentum-restricted electron-hole pairs we then select those whose electron and hole are found within a radial distance $|r - \bar{r}| \lesssim \Delta r$ of each other (in real space), as described by the reduced density matrix,

$$\rho^{\text{spin}}_{\sigma\bar{\sigma},\sigma'\bar{\sigma}'} \propto \int_0^R dr d\bar{r} e^{-(r-\bar{r})^2/2\Delta r^2} \tilde{\rho}^{\text{el-h}\alpha\bar{\alpha}}_{\sigma\bar{\sigma},\sigma'\bar{\sigma}'}(r,\bar{r};r,\bar{r}). \tag{A2}$$

Here, $\tilde{\rho}^{\text{el-h}}$ is the Fourier transform of $\rho^{\text{el-h}}$ [with the sign convention of Eq. (A5)] and $R$ is the radius of the graphene sheet. As explained in the main text, $\rho^{\text{el-h}\alpha\bar{\alpha}}$ is independent of

$\hat{\boldsymbol{\alpha}}$ and $\hat{\bar{\boldsymbol{\alpha}}}$ for the state $\psi_\zeta$ [Eq. (4)]. We have therefore suppressed these indices of $\rho^{\text{spin}}$. For the proposed Bell test one needs to measure the pseudospin correlators $C_{ab}$ [Eq. (8)] evaluated for the postselected electron-hole pairs. They follow from

$$C_{ab}^{\uparrow}=\frac{1}{4}\text{Tr }\rho^{\text{spin}}(1+\hat{\boldsymbol{a}}\cdot\boldsymbol{\sigma}^{\text{el}})(1+\hat{\boldsymbol{b}}\cdot\boldsymbol{\sigma}^{\text{h}}) \tag{A3}$$

as $C_{ab}=C_{ab}^{\uparrow}+C_{ba}^{\uparrow}-C_{-ab}^{\uparrow}-C_{b-a}^{\uparrow}-C_{a-b}^{\uparrow}-C_{-ba}^{\uparrow}+C_{-a-b}^{\uparrow}+C_{-b-a}^{\uparrow}$. We assume $\Omega\gg v/\Delta r\gg\max\{\Omega(eV_{\text{ex}}/\varepsilon_{\text{F}})^2,kT(\varepsilon_{\text{F}}/eV_{\text{ex}})^2\times(kT/\Omega)\exp(-2\omega/kT)\}$. In this limit it is very unlikely for two holes to be within a distance $\Delta r$ of the same electron (or vice versa). The correlators $C_{ab}^{\uparrow}$ may then be measured by "coincidence detection,[4,5]" correlating momentum-projected electron and hole pseudospin densities,

$$C_{ab}^{\uparrow}\int_0^R drd\bar{r}e^{-(r-\bar{r})^2/2\Delta r^2}\langle n_a^{\uparrow\text{el}}(r)n_b^{\uparrow\text{h}}(\bar{r})\rangle. \tag{A4}$$

Here, the densities $n^{\uparrow\text{el}}$ and $n^{\uparrow\text{h}}$ are defined in terms of the momentum-projected electron annihilation operators,

$$\vec{\psi}_\alpha^{\mu}(r)=\int_0^\infty\frac{dp}{2\pi}e^{ipr}\Theta[\gamma^{\mu}(vp-\varepsilon_{\text{F}})-\omega]\sum_{\eta=\pm1}\mathcal{P}_{\eta\hat{\boldsymbol{\alpha}}}\vec{\psi}_{\eta p\hat{\boldsymbol{\alpha}}}, \tag{A5}$$

as $n_\alpha^{\uparrow\mu}(r)=[\vec{\psi}_\alpha^{\mu\dagger}(r)\cdot\vec{v}_\alpha^{\uparrow}][\vec{v}_\alpha^{\uparrow\dagger}\cdot\vec{\psi}_\alpha^{\mu}(r)]$. The index $\mu$ takes the values el or h and we have introduced $\gamma^{\text{el}}=1$ and $\gamma^{\text{h}}=-1$. The vectors $v_\alpha^{\uparrow}$ are the spinors corresponding to pseudospin up along the directions $\hat{\boldsymbol{\alpha}}$ and the pseudospin matrix $\mathcal{P}_{\boldsymbol{p}}$ projects onto the conduction band. Note that at this point the choice of momentum direction of the postselected excitations is arbitrary: in the state (4) amplitudes with any momentum have the same pseudospin. Our above choice of momenta along the desired pseudospin quantization axis $\hat{\boldsymbol{\alpha}}$ has been made merely for convenience. In our limit, when all electron-hole pairs are well separated from each other (much farther than $\Delta r$), the density correlator of Eq. (A4) may be replaced by its irreducible contribution

$$C_{ab}^{\uparrow}\propto\int_0^R drd\bar{r}e^{-(r-\bar{r})^2/2\Delta r^2}\langle\delta n_a^{\uparrow\text{el}}(r)\delta n_b^{\uparrow\text{h}}(\bar{r})\rangle \tag{A6}$$

(Refs. 4 and 5). The momentum-projected density $n_\alpha^{\mu}$ is in principle experimentally accessible, for instance by tunneling electrons and holes into additional reservoirs that couple symmetrically to two contacts $\alpha$ and $-\alpha$ via momentum-dependent tunnel amplitudes $w_\alpha^{\text{el}}$ and $w_\alpha^{\text{h}}$. Pseudospin-resolved amplitudes $w^{\mu}$, as described by the tunneling Hamiltonian $H_{\text{T}\alpha}^{\text{el}}+H_{\text{T}\alpha}^{\text{h}}$ with

$$H_{\text{T}\alpha}^{\mu}=\int drw_\alpha^{\mu}(r)\vec{\psi}_\alpha^{\mu\dagger}(r)\cdot\vec{\psi}_\alpha^{\text{res}\mu}+\text{H.c.}, \tag{A7}$$

then allow to obtain the correlator $C_{ab}^{\uparrow}$ as

$$C_{ab}^{\uparrow}\propto c_{ab}^{\uparrow\text{el-h}}+\mathcal{O}(e^{-\omega/kT}) \tag{A8}$$

from current correlators,

$$c_{ab}^{\uparrow\text{el-h}}=\int dte^{-(vt)^2/2\Delta r^2}\langle\delta I_a^{\uparrow\text{el}}(t)\delta I_b^{\uparrow\text{h}}(0)\rangle, \tag{A9}$$

where

$$I_\alpha^{\uparrow\mu}=ie\int drw_\alpha^{\mu}(r)[\vec{\psi}_\alpha^{\mu\dagger}(r)\cdot\vec{v}_\alpha^{\uparrow}](\vec{v}_\alpha^{\uparrow\dagger}\cdot\vec{\psi}_\alpha^{\text{res}\mu})+\text{H.c.} \tag{A10}$$

We have introduced one reservoir for every pseudospin state, with corresponding electron annihilation operators $\psi_{\alpha\sigma}^{\text{res}\mu}$. The amplitudes $w_\alpha^{\mu}$ are peaked at the radii $r_\alpha^{\mu}$. In the step from Eq. (A6)–(A8) we have assumed time-translational invariance and $r_\alpha^{\text{el}}=r_\alpha^{\text{h}}$. Note that the error due to thermal excitations flowing from the reservoirs into the graphene sheet (the origin of the problem pointed out in Ref. 6) is here exponentially suppressed. It is of order $\exp(-\omega/kT)$ since it is only pairs of excitations that differ in energy by at least $\omega$ that contribute to the correlator $c^{\uparrow\text{el-h}}$.

The correlator $C_{ab}^{\uparrow}$, however, may be accessed also through measurements of correlators,

$$c_{ab}^{\uparrow}=\int dte^{-(vt)^2/2\Delta r^2}\langle\delta I_a^{\uparrow}(t)\delta I_b^{\uparrow}(0)\rangle, \tag{A11}$$

of currents,

$$I_\alpha^{\uparrow}=ie\int drw_\alpha(r)[\vec{\psi}_\alpha^{\dagger}(r)\cdot\vec{v}_\alpha^{\uparrow}](\vec{v}_\alpha^{\uparrow\dagger}\cdot\vec{\psi}_\alpha^{\text{res}})+\text{H.c.}, \tag{A12}$$

without energy selectivity, where $\vec{\psi}_\alpha(r)=\int_0^\infty\frac{dp}{2\pi}\exp(ipr)\Sigma_{\eta=\pm1}\mathcal{P}_{\eta\hat{\boldsymbol{\alpha}}}\vec{\psi}_{\eta p\hat{\boldsymbol{\alpha}}}$ (and a corresponding tunneling Hamiltonian $H_{\text{T}\alpha}$). This is seen easiest by separating two contributions to the above current correlators from each other; first there are contributions due to the electron-hole pairs created by $V_{\text{ex}}$. We denote these contributions to $c^{\uparrow\text{el-h}}$ and $c^{\uparrow}$ by a subscript "ex," $c_{\text{ex}}^{\uparrow\text{el-h}}$ and $c_{\text{ex}}^{\uparrow}$, respectively. Second, there are contributions from statistical correlations of excitations due to the Pauli principle, denoted by a subscript "stat," $c_{\text{stat}}^{\uparrow\text{el-h}}$ and $c_{\text{stat}}^{\uparrow}$. We have $c^{\uparrow}=c_{\text{ex}}^{\uparrow}+c_{\text{stat}}^{\uparrow}$ and likewise $c^{\uparrow\text{el-h}}=c_{\text{ex}}^{\uparrow\text{el-h}}+c_{\text{stat}}^{\uparrow\text{el-h}}$. The typical energy separation between the electrons and the holes created by $V_{\text{ex}}$ is $vp-v\bar{p}\approx\Omega$. For the currents $I_{\text{ex}\alpha}^{\uparrow}$ carried by these excitations we thus may approximate $I_{\text{ex}\alpha}^{\uparrow}=I_{\text{ex}\alpha}^{\uparrow\text{el}}+I_{\text{ex}\alpha}^{\uparrow\text{h}}+\mathcal{O}(\omega/\Omega)$ and $c_{\text{ex}ab}^{\uparrow}=c_{\text{ex}ab}^{\uparrow\text{el-h}}+c_{\text{ex}ba}^{\uparrow\text{el-h}}+\mathcal{O}(\omega/\Omega)$. Under the assumption $kT\gg v/|r_a^{\mu}\hat{\boldsymbol{a}}\pm r_b^{\mu}\hat{\boldsymbol{b}}|$ made in the main text the statistical correlations $c_{\text{stat}}^{\uparrow}$ are identical to the equilibrium correlations, $c_{\text{stat}}^{\uparrow}=c^{\uparrow}|_{V_{\text{ex}}=0}+\mathcal{O}[kT\exp(-kT|r_a^{\mu}\hat{\boldsymbol{a}}\pm r_b^{\mu}\hat{\boldsymbol{b}}|/v)]$ (see Appendix B). Moreover, statistical fluctuations do not contribute to the momentum-projected correlator by our definition of $\vec{\psi}^{\mu}$: $\langle\psi_{\alpha\sigma}^{\text{el}\dagger}(r,t)\psi_{\alpha'\sigma'}^{\text{h}}(r',t')\rangle=0$. We conclude that to leading order in our limit the momentum-projected irreducible current correlator $c^{\uparrow\text{el-h}}$ may be expressed through the corresponding correlator without momentum projection after subtraction of its statistical background, $c_{ab}^{\uparrow\text{el-h}}+c_{ba}^{\uparrow\text{el-h}}=c_{\text{ex}ab}^{\uparrow\text{el-h}}+c_{\text{ex}ba}^{\uparrow\text{el-h}}=c_{\text{ex}ab}^{\uparrow}=c_{ab}^{\uparrow}-c_{ab}^{\uparrow}|_{V_{\text{ex}}=0}$, such that

$$C^{\uparrow}_{ab} + C^{\uparrow}_{ba} \propto c_{ab} - c_{ab}|_{V_{\rm ex}=0}. \tag{A13}$$

The pseudospin-resolved tunneling amplitudes assumed above are rather unrealistic. Alternatively the pseudospin currents $I^{\uparrow}_{\alpha}$ can be measured by introducing separate reservoirs for the contacts $\alpha$ and $-\alpha$, as explained after Eq. (6) of the main text. One further shows straightforwardly along the lines of Refs. 4 and 5 that the correlators $c$, Eq. (A11) may be replaced by zero-frequency correlators [with an error of $\mathcal{O}(v/\Omega\Delta r)$]. Equations (9) and (10) then follow after a normalization of $C^{\uparrow}$ along the lines of Refs. 4 and 5. Values for $\omega$ and $\Delta r$ that satisfy all of the above conditions can be found provided that $kT \ll \Omega$. A violation of the inequality (7) with the correlators Eqs. (9) and (10) is thus indeed an entanglement witness to leading order in $v/|r^{\mu}_a \hat{\boldsymbol{a}} \pm r^{\mu}_b \hat{\boldsymbol{b}}| \ll kT \ll \Omega$.

Strictly speaking, a Bell test has to demonstrate that a violation of Eq. (7) found through the current correlation measurement described above is due to the entanglement of $\rho^{\rm spin}$ rather than the deviations of the true pseudospin correlators from Eq. (9) that appear at nonzero temperature (even though those are suppressed in our limit). Collecting all the sources of such deviations mentioned above we find that the Bell parameter $\mathcal{B}$ measured through Eq. (9) is related to the Bell parameter $\mathcal{B}^{\rm spin}$ corresponding to the actual pseudospin correlators Eqs. (8) and (A3) as

$$\begin{aligned}\mathcal{B} = {} & \mathcal{B}^{\rm spin}(\omega,\Delta r) + g_1\frac{\omega}{\Omega} + g_2\frac{v}{\Omega\Delta r} + g_3\frac{\Omega\Delta r}{v}\left(\frac{eV_{\rm ex}}{\varepsilon_{\rm F}}\right)^2 \\ & + g_4\frac{kT\Delta r}{v}\frac{kT}{\Omega}\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^2 e^{-2\omega/kT} \\ & + \sum_{\alpha\alpha'} g_{\alpha\alpha'}\frac{kT}{\Omega}\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^2 e^{-kT|r_\alpha\hat{\boldsymbol{\alpha}} - r_{\alpha'}\hat{\boldsymbol{\alpha}}'|/v},\end{aligned} \tag{A14}$$

up to terms of higher order in small quantities, with positive constants $g_j$ and $g_{\alpha\alpha'}$ that are of order unity. The summation in Eq. (A14) runs over all measured combinations of tunnel contacts $\alpha$ and $\alpha'$. Entanglement is conclusively demonstrated if one finds a violation of the inequality $\mathcal{B}^{\rm spin} \leq 2$, that is if

$$\begin{aligned}\mathcal{B} \leq {} & 2 + g_1\frac{\omega}{\Omega} + g_2\frac{v}{\Omega\Delta r} + g_3\frac{\Omega\Delta r}{v}\left(\frac{eV_{\rm ex}}{\varepsilon_{\rm F}}\right)^2 \\ & + g_4\frac{kT\Delta r}{v}\frac{kT}{\Omega}\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^2 e^{-2\omega/kT} \\ & + \sum_{\alpha\alpha'} g_{\alpha\alpha'}\frac{kT}{\Omega}\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^2 e^{-kT|r_\alpha\hat{\boldsymbol{\alpha}} - r_{\alpha'}\hat{\boldsymbol{\alpha}}'|/v}.\end{aligned} \tag{A15}$$

A direct violation of Eq. (A15) requires knowledge of the parameters $g_j$ and $g_{\alpha\alpha'}$. They can in principle be determined if the frequency spectrum of the voltage correlator $c_V$ is known. More practically, however, one may measure the dependence of $\mathcal{B}$ on parameters such as $T$ and $V_{\rm ex}$ and establish an inconsistency with Eq. (A15) for a particular choice of $\omega$ and $\Delta r$. For example, one may choose $\omega = \sqrt{kT\Omega}$ and $\Delta r = \varepsilon_{\rm F} v/\Omega eV_{\rm ex}$, such that

$$\begin{aligned}\mathcal{B} = {} & \mathcal{B}^{\rm spin}(\sqrt{kT\Omega}, \varepsilon_{\rm F}v/\Omega eV_{\rm ex}) + g_1\sqrt{\frac{kT}{\Omega}} + (g_2+g_3)\frac{eV_{\rm ex}}{\varepsilon_{\rm F}} \\ & + g_4\left(\frac{kT}{\Omega}\right)^2\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^3 e^{-2\sqrt{\Omega/kT}} \\ & + \sum_{\alpha\alpha'} g_{\alpha\alpha'}\frac{kT}{\Omega}\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^2 e^{-kT|r_\alpha\hat{\boldsymbol{\alpha}} - r_{\alpha'}\hat{\boldsymbol{\alpha}}'|/v}.\end{aligned} \tag{A16}$$

An analysis of $\rho^{\rm spin}$, Eqs. (A1) and (A2), shows for the same choice of $\omega$ and $\Delta r$,

$$\begin{aligned}& \mathcal{B}^{\rm spin}(\sqrt{kT\Omega}, \varepsilon_{\rm F}v/\Omega eV_{\rm ex}) \\ & = \mathcal{B}^{\rm spin}(\sqrt{kT_0\Omega}, \varepsilon_{\rm F}v/\Omega eV_0) + \tilde{g}_1\left(\sqrt{\frac{kT}{\Omega}} - \sqrt{\frac{kT_0}{\Omega}}\right) \\ & \quad + \tilde{g}_3\left(\frac{eV_{\rm ex}}{\varepsilon_{\rm F}} - \frac{eV_0}{\varepsilon_{\rm F}}\right) + \tilde{g}_4\left[\left(\frac{kT}{\Omega}\right)^2\left(\frac{\varepsilon_{\rm F}}{eV_{\rm ex}}\right)^3 e^{-2\sqrt{\Omega/kT}} \right. \\ & \quad \left. - \left(\frac{kT_0}{\Omega}\right)^2\left(\frac{\varepsilon_{\rm F}}{eV_0}\right)^3 e^{-2\sqrt{\Omega/kT_0}}\right],\end{aligned} \tag{A17}$$

up to terms of higher order in small quantities, with positive constants $\tilde{g}_j$ that are of order unity. Suppose that with this choice of $\omega$ and $\Delta r$ one measures a parameter $\mathcal{B}-2>0$ that varies only by a small fraction $\Delta\mathcal{B}/(\mathcal{B}-2) \ll 1$ over a range of temperatures $T \in [T_0, 2T_0]$ and potentials $|V_{\rm ex}| \in [V_0, 2V_0]$ $(V_0 > 0)$. Equations (A16) and (A17) prove this measurement result to be inconsistent with the assumption that no entanglement is present in the electron-hole pairs measured at $T_0$ and $V_0$. It excludes that $\mathcal{B}^{\rm spin}(\sqrt{kT_0\Omega}, \varepsilon_{\rm F}v/\Omega eV_0) \leq 2$ [note that according to Eqs. (A16) and (A17) all $T$- and $V_{\rm ex}$-independent contributions to $\mathcal{B}$ have a negative sign]. The result $\mathcal{B}-2>0$ with variation $\Delta\mathcal{B}/(\mathcal{B}-2) \ll 1$ over the above parameter range, however, is predicted to be found in the presence of entanglement (for appropriate choices of the pseudospin projection directions such as shown in Fig. 1). In such a measurement the generated entanglement can thus be verified conclusively.

## APPENDIX B: CURRENT CORRELATORS

In this appendix we obtain the correlators $\tilde{c}_{ab} = \int dt \langle \delta I_a(t) \delta I_b(0) \rangle$ from the Hamiltonian $H = H_0 + H_{\rm ex} + \sum_\alpha H_{{\rm T},\alpha}$. Lowest order perturbation theory in $\vec{w}$ and $V_{\rm ex}$ results in

$$\begin{aligned}c^{(0)}_{ab} = {} & -\frac{e^4}{2}\int d\boldsymbol{x}_a d\boldsymbol{x}'_a d\boldsymbol{x}_b d\boldsymbol{x}'_b \frac{d\varepsilon}{2\pi}\frac{d\varepsilon'}{2\pi} {\rm Tr}\, \check{g}(\boldsymbol{x}'_b - \boldsymbol{x}_a, \varepsilon) \\ & \times [\check{\tau}, \check{G}_a(\varepsilon)] \check{W}_a(\boldsymbol{x}_a, \boldsymbol{x}'_a) \check{g}(\boldsymbol{x}'_a - \boldsymbol{x}_b, \varepsilon') \\ & \times [\check{\tau}, \check{G}_b(\varepsilon)] \check{W}_b(\boldsymbol{x}_b, \boldsymbol{x}'_b),\end{aligned} \tag{B1}$$

describing statistical correlations. Here, $\check{g}$ and $\check{G}_\alpha$, are the Green's functions of electrons in the graphene sheet and in reservoir $\alpha$, respectively, and [,] denotes the matrix commutator. They are matrices with Keldysh[27] and pseudospin indices. We expand the Green's functions $\check{g}(\boldsymbol{x})$ asymptotically assuming $k_{\rm F}|\boldsymbol{x}| \gg 1$,[28] as it is appropriate in our limit $k_{\rm F} r_\alpha$

$\gg 1$. The matrix $\check{\tau}^z$ is the third Pauli matrix in Keldysh space and the unit matrix in pseudospin space, while $\check{W}_\alpha$ is unity in Keldysh space and

$$\check{W}_\alpha(\boldsymbol{x}_\alpha,\boldsymbol{x}'_\alpha) = \begin{pmatrix} w_\alpha^{\rm A}(\boldsymbol{x}_\alpha)w_\alpha^{\rm A*}(\boldsymbol{x}'_\alpha) & w_\alpha^{\rm A}(\boldsymbol{x}_\alpha)w_\alpha^{\rm B*}(\boldsymbol{x}'_\alpha) \\ w_\alpha^{\rm B}(\boldsymbol{x}_\alpha)w_\alpha^{\rm A*}(\boldsymbol{x}'_\alpha) & w_\alpha^{\rm B}(\boldsymbol{x}_\alpha)w_\alpha^{\rm B*}(\boldsymbol{x}'_\alpha) \end{pmatrix} \quad \text{(B2)}$$

in pseudospin space.

At the next to leading (second) order in $V_{\rm ex}$ we find a contribution,

$$\begin{aligned} c_{ab}^{(2)\rm ex} = &-\frac{e^4}{2}\int d\boldsymbol{x}_a d\boldsymbol{x}'_a d\boldsymbol{x}_b d\boldsymbol{x}'_b d\boldsymbol{x}_{\rm ex} d\boldsymbol{x}'_{\rm ex}\frac{d\varepsilon}{2\pi}\frac{d\varepsilon'}{2\pi}c_V(\varepsilon-\varepsilon') \\ &\times u(\boldsymbol{x}_{\rm ex})u(\boldsymbol{x}'_{\rm ex})\,\mathrm{Tr}\;\check{g}(\boldsymbol{x}'_b-\boldsymbol{x}_{\rm ex},\varepsilon)\check{\tau}^z\check{g}(\boldsymbol{x}_{\rm ex}-\boldsymbol{x}_a,\varepsilon') \\ &\times[\check{\tau}^z,\check{G}_a(\varepsilon')]\check{W}_a(\boldsymbol{x}_a,\boldsymbol{x}'_a)\check{g}(\boldsymbol{x}'_a-\boldsymbol{x}'_{\rm ex},\varepsilon')\check{\tau}^z\check{g}(\boldsymbol{x}'_{\rm ex}-\boldsymbol{x}_b,\varepsilon) \\ &\times[\check{\tau}^z,\check{G}_b(\varepsilon)]\check{W}_b(\boldsymbol{x}_b,\boldsymbol{x}'_b), \end{aligned} \quad \text{(B3)}$$

due to electron-hole pairs excited by the fluctuating potential $V_{\rm ex}$ and a contribution,

$$\begin{aligned} c_{ab}^{(2)\rm stat} = &-\frac{e^4}{2}\int d\boldsymbol{x}_a d\boldsymbol{x}'_a d\boldsymbol{x}_b d\boldsymbol{x}'_b d\boldsymbol{x}_{\rm ex} d\boldsymbol{x}'_{\rm ex}\frac{d\varepsilon}{2\pi}\frac{d\varepsilon'}{2\pi}c_V(\varepsilon-\varepsilon') \\ &\times u(\boldsymbol{x}_{\rm ex})u(\boldsymbol{x}'_{\rm ex})\mathrm{Tr}\{\check{g}(\boldsymbol{x}'_b-\boldsymbol{x}_{\rm ex},\varepsilon)\check{\tau}^z \\ &\times\check{g}(\boldsymbol{x}_{\rm ex}-\boldsymbol{x}'_{\rm ex},\varepsilon')\check{\tau}^z\check{g}(\boldsymbol{x}'_{\rm ex}-\boldsymbol{x}_a,\varepsilon) \\ &\times[\check{\tau}^z,\check{G}_a(\varepsilon)]\check{W}_a(\boldsymbol{x}_a,\boldsymbol{x}'_a)\check{g}(\boldsymbol{x}'_a-\boldsymbol{x}_b,\varepsilon) \\ &\times[\check{\tau}^z,\check{G}_b(\varepsilon)]\check{W}_b(\boldsymbol{x}_b,\boldsymbol{x}'_b)+\check{g}(\boldsymbol{x}'_b-\boldsymbol{x}_a,\varepsilon) \\ &\times[\check{\tau}^z,\check{G}_a(\varepsilon)]\check{W}_a(\boldsymbol{x}_a,\boldsymbol{x}'_a)\check{g}(\boldsymbol{x}'_a-\boldsymbol{x}_{\rm ex},\varepsilon)\check{\tau}^z \\ &\times\check{g}(\boldsymbol{x}_{\rm ex}-\boldsymbol{x}'_{\rm ex},\varepsilon')\check{\tau}^z\check{g}(\boldsymbol{x}'_{\rm ex}-\boldsymbol{x}_b,\varepsilon) \\ &\times[\check{\tau}^z,\check{G}_b(\varepsilon)]\check{W}_b(\boldsymbol{x}_b,\boldsymbol{x}'_b)\}, \end{aligned} \quad \text{(B4)}$$

which describes statistical correlations due to a renormalization of the tunneling amplitudes $\vec{w}$ by $V_{\rm ex}$.

To lowest order in our limit $\Omega \ll \varepsilon_{\rm F}$ and $l_{\rm ex}, l_\alpha \ll |\boldsymbol{r}_\alpha|$ we have $w_\alpha^\sigma(\boldsymbol{x}) = (v/\sqrt{k_{\rm F}})\bar{w}_\alpha^\sigma\delta(\boldsymbol{x}-r_\alpha\hat{\boldsymbol{\alpha}})$ and $u(\boldsymbol{x}) = \delta(\boldsymbol{x})/k_{\rm F}^2$. Equation (B3) then evaluates to

$$\tilde{c}_{a,b}^{(2)\rm ex} = \frac{e^4}{2\varepsilon_{\rm F}^2}(1+\hat{\boldsymbol{a}}\cdot\hat{\boldsymbol{b}})W_aW_b\int_{-\infty}^{\varepsilon_{\rm F}}d\varepsilon\int_{\varepsilon_{\rm F}}^{\infty}d\varepsilon'\,c_V(\varepsilon-\varepsilon'), \quad \text{(B5)}$$

with the tunneling probabilities,

$$W_\alpha = \varepsilon_{\rm F}\frac{|\bar{w}_\alpha^{\rm A}|^2+|\bar{w}_\alpha^{\rm B}|^2-2|\bar{w}_\alpha^{\rm A}\bar{w}_\alpha^{\rm B}|(\cos\nu_\alpha,\sin\nu_\alpha)\cdot\hat{\alpha}}{4\pi^2 v_\alpha k_{\rm F}^2 r_\alpha}, \quad \text{(B6)}$$

where $\nu_\alpha = i\ln(\bar{w}_\alpha^{\rm A}\bar{w}_\alpha^{\rm B*}/|\bar{w}_\alpha^{\rm A}\bar{w}_\alpha^{\rm B}|)$ and $v_\alpha$ is the Fermi velocity in reservoir $\alpha$. The above perturbation expansion in $\vec{w}$ is justified if $\bar{w} \ll 1$.

The contribution $c^{(2)\rm stat}$ [Eq. (B4)] is exponentially suppressed as $\exp(-kT|r_a\hat{\boldsymbol{a}}\pm r_b\hat{\boldsymbol{b}}|/v)$ by thermal dephasing in the limit $kT \gg v/|r_a\hat{\boldsymbol{a}}\pm r_b\hat{\boldsymbol{b}}|$ taken in the main text. After the subtraction of the statistical fluctuations $c^{(0)}$ and the normalization performed in Eq. (10) the correlator $c_{a,b}$ takes the form of Eq. (12) to this accuracy. Similarly we obtain for the ac current into tunnel contact $\alpha$,

$$\begin{aligned} I_\alpha(\omega) = ie^2V_{\rm ex}(\omega)\int &d\boldsymbol{x}_\alpha d\boldsymbol{x}'_\alpha d\boldsymbol{x}_{\rm ex}\frac{d\varepsilon}{2\pi}u(\boldsymbol{x}_{\rm ex}) \\ &\times\mathrm{Tr}\;\check{g}(\boldsymbol{x}'_\alpha-\boldsymbol{x}_{\rm ex},\varepsilon)\check{\tau}^z\check{g}(\boldsymbol{x}_{\rm ex}-\boldsymbol{x}_\alpha,\varepsilon-\omega) \\ &\times[\check{\tau}^z\check{G}_\alpha(\varepsilon)-\check{G}_\alpha(\varepsilon-\omega)\check{\tau}^z]\check{W}_\alpha(\boldsymbol{x}_\alpha,\boldsymbol{x}'_\alpha), \end{aligned} \quad \text{(B7)}$$

which implies Eq. (11) in our limit.


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[16] The single-valley form of $H_{{\rm T},j}$ exploits the time-reversal invariance of the tunneling Hamiltonian (at $B=0$). The spinor $\vec{w}_j$ is obtained from the corresponding spinor $(\tilde{w}_j^{\rm A},\tilde{w}_j^{\rm B},\tilde{w}_j'^{\rm A},\tilde{w}_j'^{\rm B})$ in the original two-valley model by $w_j^{\rm A} = [\tilde{w}_j^{\rm A}+(\tilde{w}_j'^{\rm A})^*]/\sqrt{2}a$ and $w_j^{\rm B} = [\tilde{w}_j^{\rm B}-(\tilde{w}_j'^{\rm B})^*]/\sqrt{2}$.
[17] A rigorous quantum-mechanical treatment of the problem that projects the Bell pair wave function Eq. (4) onto the eigenstates of $\boldsymbol{x}(t)$ [Eq. (6)] to leading order in $(k_{\rm F}r_j)^{-1}$ confirms this semiclassical reasoning. In momentum space and polar coordinates

the eigenstates to eigenvalue $\boldsymbol{r}=(r,\phi)$ take the form $\psi_{\pm}(p,\theta) = e^{\pm ivtp - ipr\cos(\phi-\theta)}(1, \pm e^{i\theta})$ to this accuracy.
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